CHAPTER – 5

TIME AND DISTANCE

In this chapter, we will look at problems in the following different areas:

- General problems on Time, Speed and Distance
- Relative Speed
- Boats and Streams
- Races and Circular Tracks

Before we look at problems in various areas, let us first look at some basic concepts pertaining to speed, time and distance.

SPEED

Distance covered per unit time is called speed.
i.e., Speed = Distance/time

The above relationship between the three variables distance, speed and time can also be expressed as follows:

Distance = Speed $\times$ Time or
Time = Distance/Speed

- If two bodies travel with the same speed, Distance covered α Time (Direct Variation).
- If two bodies travel for the same period of time, Distance covered α Speed (Direct Variation).
- If two bodies travel the same distance,

  Time $\alpha \frac{1}{\text{Speed}}$ (Inverse Variation)

Distance is normally measured in kilometres, metres or miles; time in hours or seconds and speed in km/hr (also denoted by kmph), miles/hr (also denoted by mph) or metres/second (denoted by m/s).

To convert speed in kmph to m/sec, multiply it with 5/18.

To convert speed in m/sec to kmph, multiply it with 18/5.

In the case of moving trains, three different situations need to be considered.

When a train passes a stationary point, the distance covered (in the passing) is the length of the train. If the train is crossing a platform (or a bridge), the distance covered by the train (in the crossing) is equal to the length of the train plus the length of the platform (or bridge). If two trains pass each other (travelling in the same direction or in opposite directions), the total distance covered (in the crossing or the overtaking, as the case may be) is equal to the sum of the lengths of the two trains.

AVERAGE SPEED

Average speed of a body travelling at different speeds is defined as follows :

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Please note that the **AVERAGE SPEED** of a body is **NOT** always equal to the **AVERAGE OF THE SPEEDS.**

If a body travels from point A to point B with a speed of p and back to point A (from point B) with a speed of q, then the average speed of the body can be calculated as **2pq/(p + q)**. Please note that this does not depend on the distance between A and B.

If a body covers part of the journey at speed p and the remaining part of the journey at speed q and the distances of the two parts of the journey are in the ratio m : n, then the average speed for the entire journey is **(m + n)pq/(mq + np)**.

Examples

5.01. Express a speed of 20 m/s in kmph?

Sol: Speed = $20 \times \frac{18}{5}$ = 72 km/hr

5.02. A car can cover 600 km in 8 hours. If the speed is increased by 25 kmph, how much time does the car take to cover a distance of 800 km?

Sol: Speed = $\frac{\text{Distance}}{\text{Time}} = \frac{600}{8}$ = 75 kmph

Now this is increased by 25 kmph. Hence new speed is 100 kmph. At this speed, time taken to cover 800 km = 800/100
= 8 hours

5.03. A person covers a certain distance at a certain speed. If he decreases his speed by 20%, then he takes 16 minutes more to cover the same distance. Find the time taken by him to cover the distance at original speed.

Sol: When the speed decreases by 20%, the new speed is 80% of the original speed, it is (4/5) times the original speed. Since speed and time are inversely related, if the speed is (4/5) times the original speed, then the time will be (5/4) times the original time. Let the time taken to cover the distance at the original speed be t.

Then, $\frac{5}{4}t - t = \frac{1}{4}t = 16$

$\Rightarrow$ t = 4 x 16 = 64 minutes.

5.04. A car covers a certain distance at a speed of 90 km/hr while going and returns to the starting point at a speed of 60 km/hr. Find the average speed of the car for the whole journey.

Sol: We know that the average speed is 2pq/(p + q) where p and q are the speeds for covering two equal distances.
Therefore the average speed
= $(2 \times 90 \times 60)/(90 + 60)$ = 72 km/hr

5.05. What is the time taken by a train of length 360 m to cross a pole at a speed of 72 kmph?

Sol: Time taken by the train to cross the pole

$= \frac{\text{Length of the train}}{\text{Speed of the train}}$

$= \frac{360}{72 \times (5/18)} = \frac{360}{20} = 18$ seconds

5.06. How long will a train 200 m long travelling at a speed of 54 kmph take to cross a platform of length 100 m?

Sol: Distance covered by the train
= Length of the train + length of the platform = 200 + 100 = 300 m
Speed of the train
= 54 kmph = 54 $\times$ (5/18) = 15 m/sec
$\therefore$ Time taken by the train
= 300/15 = 20 seconds

5.07. Find the length of a train running at 45 kmph which can cross a bridge of length 250 m in 36 seconds.

Sol: Speed of the train = 45 kmph
= 45 $\times$ (5/18) = 12.5 m/s
Distance covered by the train = 12.5 $\times$ 36
= 450 m.
Length of the train = Distance covered – length of the bridge = 450 – 250 = 200 m

5.08. A worker reaches his work place 20 minutes late by walking at 3 km/hr. The next day he increases his speed by 2 km/hr and reaches on time. Find the distance from his house to his work place.

Sol: Let the distance be x km. Then, time taken on the 1st day = x/3 hrs
Time taken on the 2nd day = x/5 hrs
Given that the travel time on the 1st day is 20 minutes more than that on the 2nd day.

$$\frac{x}{3} - \frac{x}{5} = \frac{20}{60} \Rightarrow x = 2.5$$

> In general, if a person travelling between two points reaches p hours late travelling at a speed of u kmph and reaches q hours early travelling at v kmph, the distance between the two points is given by $\frac{\mathbf{vu}}{\mathbf{v-u}}\mathbf{(p+q)}$.

5.09. A person travelling at 6 kmph reaches his office 15 minutes late. Had he travelled at 8 kmph he would have been 25 minutes early. Find the distance the person has to travel to reach his office.

Sol: **Method 1**

Using the formula, $d = \frac{vu}{v-u}(p+q)$,

$$\text{distance} = \frac{6 \times 8}{8-6}\left(\frac{15}{60} + \frac{25}{60}\right) = 16 \text{ km}$$

Method 2
Let the distance be d km.
Difference of the travel times when travelling at 6 kmph and 8 kmph is 40 minutes.

$$\frac{d}{6} - \frac{d}{8} = \frac{40}{60}$$

$$\frac{d}{24} = \frac{40}{60}$$

d = 16

RELATIVE SPEED

The speed of one (moving) body in relation to another moving body is called the relative speed of these two bodies, i.e., it is the speed of one moving body as observed, from the second moving body.

If two bodies are moving in the same direction, the relative speed is equal to the difference of the speeds of the two bodies.

If two bodies are moving in opposite directions, the relative speed is equal to the sum of the speeds of the two bodies.

5.10. Find the time taken by a train of length 100 m running at a speed of 72 kmph to cross another train of length 200 m running at a speed of 63 kmph in the same direction.

Sol: Total distance covered = sum of the lengths of the two trains = 100 + 200
= 300 m.
Relative speed of the two trains = 72 – 63 = 9 kmph
(Since the trains are running in the same direction, the relative speed will be the difference of the speeds)

$$= 9 \times \frac{5}{18} = \frac{5}{2} \text{m/s}$$

Therefore time taken
= 300/(5/2) = 120 seconds

5.11. A train crosses two cyclists travelling in the same direction as the train, in 12 seconds and 15 seconds respectively. If the speeds of the two cyclists are 10 kmph and 12 kmph respectively, then find the length and the speed of the train.

Sol: Let the speed of the train be s kmph.
Relative speed in overtaking the first cyclist
= (s – 10) kmph. Time taken to overtake the first cyclist = 12 seconds.
Hence the length of the train
= (12) (s – 10) (5/18) $\rightarrow$ (1)
Similarly, considering the case of the overtaking the second cyclist, length of the train
= (15) (s – 12) (5/18) $\rightarrow$ (2)
Equating (1) and (2),
(12) (s – 10) (5/18) = (15) (s – 12) (5/18)
$\Rightarrow$ 12s – 120 = 15s – 180
60 = 3s $\Rightarrow$ s = 20
Length of the train = (12) (s – 10) (5/18)
= (12) (10) (5/18) = (100/3) metres

5.12. Two trains running at 63 kmph and 45 kmph cross each other in 15 seconds, when they run in opposite directions. When they run in the same direction, a person in the faster train observes that he crossed the other train in 40 seconds. Find the lengths of the two trains.

Sol: Let p and q be the lengths of the slower and faster trains respectively. When the trains are travelling in the opposite directions, relative speed = 63 + 45 = 108 kmph = 30 m/sec.
Distance covered = sum of the lengths of the two trains

$\Rightarrow$ p + q = 30 x 15 = 450 m $\rightarrow$ (1)
When trains are travelling in the same direction, since we are given that the time noted by a person in the faster train as 40 seconds to cross the slower train, the distance covered is equal to the length of the slower train, distance covered = q
Relative speed = 63 – 45 = 18 kmph
= 5 m/sec
q = (5) (40) = 200 m $\rightarrow$ (2)
From (1) and (2), we get
p = 450 – q = 250 m

5.13. Two trains of length 100 m and 250 m run on parallel lines. When they run in the same direction, it will take 70 seconds for the faster train to cross the slower train and when they run in opposite directions, they take 10 seconds to cross each other. Find the speeds of the two trains.

Sol: Let the speeds of the two trains be p and q with p > q.
Total distance covered = sum of the lengths of the two trains = 100 + 250
= 350 m.
When they run in the same direction, relative speed is given by
p – q = 350/70 = 5 $\rightarrow$ (1)
When they run in opposite direction relative speed is given by
p + q = 350/10 = 35 $\rightarrow$ (2)
Solving (1) and (2)
we get, p = 20 m/s and q = 15 m/s.

BOATS AND STREAMS

Problems related to boats and streams are different in the computation of relative speed from those of trains/cars.

When a boat is moving in the same direction as the stream or water current, the boat is said to be moving **WITH THE STREAM OR CURRENT.**

When a boat is moving in a direction opposite to that of the stream or water current, it is said to be moving **AGAINST THE STREAM OR CURRENT.**

If the boat is moving with a certain speed in water that is not moving, the speed of the boat is then called the **SPEED OF THE BOAT IN STILL WATER.**

When the boat is moving upstream, the speed of the water opposes (and hence reduces) the speed of the boat.

When the boat is moving downstream, the speed of the water aids (and thus adds to) the speed of the boat. Thus, we have

Speed of the boat against stream
= Speed of the boat in still water – Speed of the stream
Speed of the boat with the stream
= Speed of the boat in still water + Speed of the stream

These two speeds, the speed of the boat against the stream and the speed of the boat with the stream, are RELATIVE SPEEDS.

If u is the speed of the boat down the stream and v is the speed of the boat up the stream, then we have the following two relationships.

Speed of the boat in still water = (u + v)/2
Speed of the water current = (u – v)/2

In problems, instead of a boat, it may be a swimmer but the approach is exactly the same. Instead of boats/swimmers in water, it could also be a cyclist cycling against or along the wind. In some problems it can be person(s) going up/down an ascending/descending escalator. The approach to solving the problems still remains the same.

5.14. A boat travels 36 km upstream in 9 hours and 42 km downstream in 7 hours. Find the speed of the boat in still water and the speed of the water current.

Sol: Upstream speed = 36/9 = 4 kmph
Downstream speed = 42/7 = 6 kmph
Speed of the boat in still water
= (4 + 6)/2 = 5 kmph
Speed of the water current = (6 – 4)/2 = 1 kmph

5.15. A man can row at 10 kmph in still water. If it takes a total of 5 hours for him to go to a place 24 km away and return, then find the speed of the water current.

Sol: Let the speed of the water current be y kmph.
Upstream speed = (10 – y) kmph
Downstream speed = (10 + y) kmph

Total time = $\frac{24}{10+y}+\frac{24}{10-y}=5$

Hence $\frac{480}{100-y^2}=5 \Rightarrow y^2=4$

Thus y = 2

5.16. A man rows a distance of 12 km in 1 hour in still water and in 80 minutes against the current. Find the time taken by him to row 45 km with the current and return to his starting point.

Sol: Speed in still water = 12/1 = 12 kmph
Speed against the current = 12/(4/3) ($\because$ 80 min = 4/3 hrs) = 9 kmph
Therefore the speed of water current
= 12 – 9 = 3 kmph
Therefore the speed with the current
= 12 + 3 = 15 kmph
Hence time taken to row 45 km with the current and return to his starting point

$= \frac{45}{15}+\frac{45}{9}=8\,\text{hours}$

5.17. In a given time, a boat can travel down the stream a distance, which is $2^1/_2$ times that of the upstream travel. If the speed of the boat in still water is 14 kmph, find the speed of the stream.

Sol: If the distance covered down the stream is $2^1/_2$ times that covered up the stream, then the speed down the steam will also be $2^1/_2$ times the speed up the stream.
Let the speed of the stream = v
We get $\frac{14+v}{14-v}=\frac{5}{2}\Rightarrow$ v = 6 kmph

5.18. A man can row 3/7th of a kilometre upstream in 36 minutes and return in 12 minutes. Find the speed of the water current.

Sol: Upstream speed = (3/7)/(36/60) = (5/7) kmph
Downstream speed = (3/7)/(12/60) = (15/7) kmph
Speed of the water current
$= (1/2)\left[(15/7)-(5/7)\right] = (5/7)$ kmph

5.19. An escalator is moving downwards. Sam takes 30 steps to reach the bottom of the escalator from the top and John takes 70 steps to reach the top from the bottom. The speeds of Sam and John are equal. What is the number of steps in the escalator when it is stationary?

Sol: Let e be the number of steps the escalator moved for ten steps of Sam or John.
Let the number of steps in the escalator when it is stationary be N = 30 + 3e
Also, total steps = 70 – 7e
$\Rightarrow$ 30 + 3e = 70 – 7e
$\Rightarrow$ 10e = 40 $\Rightarrow$ e = 4
N = 30 + 3 $\times$ e = 42.

5.20. A takes 60 steps to reach the bottom of an escalator which is moving upwards, but B takes 30 steps to reach the top of the escalator. If A takes two steps for every step of B, then find the number of steps in the escalator when it is stationary?

Sol: Let the escalator move 'e' steps when A took 60 steps (or) B took 30 steps $\therefore$ Number of steps in the escalator = 60 – e.
Also, the number of steps in the escalator = 30 + e
$\therefore$ 60 – e = 30 + e $\Rightarrow$ 2e = 30 $\Rightarrow$ e = 15
$\therefore$ Number of steps = 30 + e = 30 + 15 = 45.

RACES AND CIRCULAR TRACKS

When two persons P and Q are running a race, they can start the race at the same time or one of them may start a little later than the other. In the second case, suppose P starts the race and after 5 seconds, Q starts. Then we say P has a "start" of 5 seconds. Alternatively, in a race between P and Q, P starts first and then when P has covered a distance of 10 metres, Q starts. Then we say that P has a "start" of 10 metres.

In a race between P and Q where Q is the winner, by the time Q reaches the winning post, if P still has another 15 metres to reach the winning post, then we say that Q has won the race by 15 metres. Similarly, if P reaches the winning post 10 seconds after Q reaches it, then we say that Q has won the race by 10 seconds.

In problems on RACES, we normally consider a 100 m race or a 1 km race. The length of the track NEED NOT necessarily be one of the two figures mentioned above but can be as given in the problem.

When two or more persons running around a circular track (starting at the same point and at the same time), then we will be interested in two main issues:
- When they will meet for the first time and
- When they will meet for the first time at the starting point

To solve the problems on circular tracks, you should keep the following points in mind.
When two persons are running around a circular track in **OPPOSITE** directions

- the relative speed is equal to the sum of the speeds of the two individuals and
- from one meeting point to the next meeting point, the two of them TOGETHER cover a distance equal to the length of the track.

When two persons are running around a circular track in the **SAME** direction

- the relative speed is equal to the difference of the speeds of the two individuals and
- from one meeting point to the next meeting point, the faster person covers one COMPLETE ROUND more than the slower person.

We can now tabulate the time taken by the persons to meet for the first time ever or for the first time at the starting point in various cases.

<u>When TWO people are running around a circular track</u>

Let the two people A and B with respective speeds of a and b (a > b) be running around a circular track (of length L) starting at the same point and at the same time. Then,

	When the two persons are running in the SAME direction	When the two persons are running in OPPOSITE directions
Time taken to meet for the FIRST TIME EVER	$\frac{L}{(a-b)}$	$\frac{L}{(a+b)}$
Time taken to meet for the first time at the STARTING POINT	LCM of $\left\{\frac{L}{a},\frac{L}{b}\right\}$	LCM of $\left\{\frac{L}{a},\frac{L}{b}\right\}$

Please note that when we have to find out the time taken by the two persons to meet for the first time at the starting point, what we have to do is to find out the time taken by each of them to complete one full round and then take the LCM of these two timings (L/a and L/b are the timings taken by the two of them respectively to complete on full round).

<u>When THREE people are running around a circular track</u>

Let the three people A, B and C with respective speeds of a, b and c (a > b > c)) be running around a circular track (of length L) starting at the same point at the same time. In this case we consider the three persons running in the same direction as the general case.

Time taken to meet for the FIRST TIME EVER	LCM of $\left\{\frac{L}{(a-b)}, \frac{L}{(b-c)}\right\}$
Time taken to meet for the first time at the STARTING POINT	LCM of $\left\{\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right\}$

The logic in obtaining the above is as follows:

A and B will be together with a time gap of L/(a – b); B and C will be together with a time gap of L/(b – c); for A, B and C to be together, A and B should be together as well as B and C should be together. Hence the LCM of the two timings L/(a – b) and L/(b – c) will give the time when A, B and C will all be together.

When we have to find out the time taken by the three persons to meet for the first time at the starting point, what we have to do is to find out the time taken by each of them to complete one full round and then take the LCM of these three timings (L/a, L/b and L/c are the timings taken by the three of them respectively to complete one full round).

Even if we are given a case where three persons are running around a circular track with two persons running in the same direction and the third in the opposite direction, we can work out the time taken by them to meet for the first time ever and for the first time at the starting point by extending the above logic.

5.21. Ajay runs $(5/3)$ times as fast as Arjun. In a race, Ajay beats Arjun by 40 meters. Find the length of the race.

Sol: Since Ajay runs $(5/3)$ times as fast as Arjun, if the length of the race is l meters

$\frac{\ell}{\ell-40}=\frac{5}{3}$

$\Rightarrow 3\ell = 5\ell - 200 \Rightarrow \ell = 100$m

Hence the length of the race is 100 m

5.22. In a 1600 m race, A beats B by 160 m and in the same race, A beats C by 340 m. By what distance does B beat C in the same race?

Sol: In the time A runs 1600 m,
B runs 1600 – 160 = 1440 m
and C runs 1600 – 340 = 1260 m
When B runs 1600 m,
C would have run $\frac{1260 \times 1600}{1440}$ =1400 m
Hence B beats C by 1600 – 1400 = 200 m

5.23. In a race of 100 m, A beats B by 40 m or 10 seconds. Find the time taken by A to run, the race and the speed of B.

Sol: Since A beats B by 40 meters, it means by the time A reaches the winning point, B is 40 m away and as A beats B by 10 seconds, it means B takes 10 seconds more time than A to reach the winning point.
Hence B covers 40 m in 10 seconds
i.e., B's speed is (40 / 10) = 4 m/s
Time taken by B to run the race
= 100 / 4 = 25 seconds
Time taken by A to run the race
= 25 – 10 = 15 seconds

5.24. In a 750 m race, the ratio of speeds of A and B is 2 : 3. B gives A a head start of 150 m. Who wins the race and by what distance?

Sol: Since A has a start of 150 m at the time B starts at the starting point, A is already 150 m ahead and hence he has only 750 – 150 = 600 m to cover.
In the time B covers 750 meters, A would have covered (2/3) × 750 = 500 m. Hence he would be 100 m from the finishing point.
Hence B wins the race and at the time he finishes the race the distance between him and A would be 100 metres

5.25. In a circular race along a track of length 3600 m, A and B start from the same point and at the same time with speeds of 36 km/hr and 45 km/hr. Find when they will meet for the first time on the track when they are running
(i) in the same direction.
(ii) in opposite directions.

Sol: Length of the track (L) = 3600 m
Speed of A = $36 \times 5/18 = 10$m/s
Speed of B = $45 \times 5/18 = 12.5$m/s
(i) Same direction :

$\text{Time} = \frac{L}{\text{Relative speed}}$

$= \frac{3600}{(12.5-10)}$ = 1440 seconds

(ii) Opposite direction :

$\text{Time} = \frac{L}{\text{Relative speed}}$

$= \frac{3600}{(12.5+10)} = \frac{3600}{22.5}$ = 160 seconds

5.26. In a circular race along a track of length 3600 m, A and B run with speeds of 27 km/hr and 36 km/hr respectively starting at the same time from the same point. When will they meet for the first time at the starting point, if they run
(i) in the same direction.
(ii) in the opposite directions.

Sol: L = 3600 m
Speed of A = $27 \times 5/18 = 7.5$m/sec
Speed of B = $36 \times 5/18 = 10$m/sec
Time taken by A to complete one round

$= \frac{3600}{7.5}$=480 sec

Time taken by B to complete one round

$= \frac{3600}{10} = 360$ sec

(i) Same direction
They will meet at the starting point at a time which is the LCM of the timings taken by each of them to complete one full round i.e., the LCM of 480 sec and 360 sec which is 1440 sec.
(ii) Opposite direction
They will meet at the starting point at a time which is the LCM of the timings taken by each of them to complete one full round i.e., the LCM of 480 seconds and 360 seconds which is 1440 seconds.

(Please note that the time taken by them to meet at the starting point does not change in the two cases i.e., it does not depend on whether the persons are running in the same direction or in opposite directions).

5.27. A, B and C with respective speeds of 18 km/hr, 27 km/hr and 36 km/hr run along a circular track 3600 m long. They start at the same time from the same point and run in the same direction. When will they meet for the first time?

Sol: L = 3600 m

Speed of A (a) = $18 \times 5/18 = 5\,m/s$

Speed of B (b) = $27 \times 5/18 = 7.5\ m/s$

Speed of C (c) = $36 \times 5/18 = 10\,m/s$

They will meet for the first time at a time which is the LCM of $\frac{L}{b-a}$ and $\frac{L}{c-b}$

$\frac{L}{b-a} = \frac{3600}{7.5-5} = 1440\,seconds$

$\frac{L}{c-b} = \frac{3600}{10-7.5} = 1140$ seconds

Therefore they will meet for the first time after 1440 seconds i.e., 24 minutes after they start.

5.28. P, Q and R run along a circular track 3600 m long at respective speeds of 18 km/hr, 27 km/hr and 36 km/hr. If they start at the same point and at the same time in the same direction when will they meet again at the starting point?

Sol: L = 3600 m

Speed of P (p) = $18 \times 5/18 = 5\ m/s$

Speed of Q (q) = $27 \times 5/18 = 7.5\,m/s$

Speed of R (r) = $36 \times 5/18 = 10\,m/s$

They will meet for the first time at the starting point after a time interval which is the LCM of $\frac{L}{p}, \frac{L}{q}$ and $\frac{L}{r}$

$\frac{L}{p} = \frac{3600}{5} = 720\,seconds$

$\frac{L}{q} = \frac{3600}{7.5} = 480\,seconds$

$\frac{L}{r} = \frac{3600}{10} = 360\,seconds$

LCM of $\frac{L}{p}, \frac{L}{q}$ and $\frac{L}{r}$ is 1440 seconds.

Hence they meet at the starting point for the first time after 1440 seconds i.e., 24 minutes from the time they start.

Note : The time taken by two or more person to meet at the starting point for the first time is not necessarily the some as the time taken by them to meet anywhere along the circular track for the first time.

CLOCKS

The hours and minutes hands of a clock move in relation to each other continuously and at any given point of time, they make an angle between 0° and 180° with each other.

If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

When we say angle between the hands, we normally refer to the acute/obtuse angles (upto 180°) between the two hands and not the reflex angle (> 180).

For solving the problems on clocks, the following points will be helpful.

Minutes hand covers 360° in 1 hour, i.e., in 60 mins. Hence **MINUTES HAND COVERS 6° PER MINUTE.**

Hours hand covers 360° in 12 hours. Hence **HOURS HAND COVERS 30° PER HOUR** or **HOURS HAND COVERS 1/2° PER MINUTE**

All angles are measured in the clockwise direction starting from the vertical line at 12 O' clock.

Note : We can also solve the problems on clocks by the route of "Relative Speed"

In 1 minute, Minutes hand covers 6° and Hours hand covers 1/2°.
Therefore, Relative Speed = 6 – 1/2
= 5½° per minute.

Alternately, in 1 hour, minutes hand covers 60 minute divisions whereas hours hand covers

∴ Relative Speed = 60 – 5 = 55 minute divisions per hour.

However, taking the route of actual angles covered is by far the simplest and easy to understand as well as helpful in solving ALL the different models of problems on CLOCKS. Hence, we will look at mainly that method only and not the relative speed method. This will not create any confusion.

The following additional points should also be remembered.

In general, every angle is made TWICE in ONE HOUR by the two hands of the clock.

In a period of 12 hours, the hands make an angle of

- 0° with each other (i.e., they coincide with each other) 11 times and hence the time gap between two successive coincidences is 12/11 hours, i.e., $1^1/_{11}$ hours, i.e., $65^5/_{11}$ minutes.
- 180° with each other (i.e., they lie on the same straight line) 11 times.
- 90° or any other angle with each other 22 times.

5.29. Find the angle between the hands of the clock when the time is 4:40 pm.

Sol: At exactly 4:00 pm the minute hand is at the same position as it would be at when the time is 12 O' clock.

From then till 4:40 pm, it travels for 40 minutes. Since the minute hand travels 6° every minute, it covers 6° × 40 = 240° in 40 minutes. At exactly 4 O' clock the hour hand makes 120° with the minute hand. We know that the hour hand covers 1/2° per minute. Hence in 40 minutes it covers 20°. So at 4:40 pm, the hour hand covers an angle of 120° + 20° = 140° from 12:00.

Hence the angle between the minute hand and the hour hand = 240° – 140° = 100°.

5.30. At what time between 5 O' clock and 6 O' clock will the two hands of the clock be at an angle of 30° with each other?

Sol: Let the time be P minutes after 5 O' clock when the hands make an angle of 30°.
The angle made by the hour hand with the vertical line (12 O' clock position of the hands) when the time is 5 O' clock is 150°. Since P minutes later the hour hand would have covered $\frac{P^\circ}{2}$ additionally, at P minutes after 5 O' clock, the hour hand makes an angle of $\left(150 + \frac{P}{2}\right)^\circ$ with the vertical line. The angle made by the minute hand with the vertical line when the time is P minutes after 5 O' clock is 6P°. For the angle between the two hands to be 30°, the difference between the two angles considered above should be equal to 30°.
Thus we get,

$6P - \left(150 + \frac{P}{2}\right) = 30$ ---------- (1) or

$\left(150 + \frac{P}{2}\right) - 6P = 30$ ---------- (2)

From (1) and (2) we get $P = 32\frac{8}{11}$ min or $P = 21\frac{9}{11}$ min. Hence the angle between the two hands of the clock will be 30°at $5 : 32\ \frac{8}{11}$ O' clock and $5 : 21\frac{9}{11}$ O' clock.

5.31. Find the time at which the hands of a clock are at right angles between 2 O' clock and 3 O' clock.

Sol: Let the time be P minutes after 2 O' clock where the angle is 90°. Then we have two equations as we saw in the previous example, (at exactly 2 O' clock the hour hand makes 60° with the vertical line (12 O' clock position of the hands))

$6P - \left(60 + \frac{P}{2}\right) = 90$ ---------- (1)

$\left(60 + \frac{P}{2}\right) - 6P = 90$ ---------- (2)

From (1) we get $P = 27\frac{3}{11}$
From (2), 60 – 90 = –30
As the position of (–30°) is same as (+330°) (i.e., – 30° + 360°)
The equation becomes, + 330° = (11p/2),
$\Rightarrow p = 60$.
This implies that at 2 hours + 60 minutes, which is same as 3 O' clock the angle is 90°.
As the question asks the time between 2 O' clock and 3 O' clock, at only $27\frac{3}{11}$ minutes past 2 O' clock will the hands of the clock be at right angles.

5.32. Find the time at which the hands of a clock are exactly in opposite directions between 9 O' clock and 10 O' clock.

Sol: The hands are in opposite directions when the angle between them is 180°. Let thehands be in opposite directions P minutes after 9 O' clock. We have the angle made by the hour hand with the vertical line (12 O' clock position of the hands) to be 270°at 9 O' clock.

Hence $\left(270 + \frac{P}{2}\right) - 6P = 180$ ---------- (1)

$6P - \left(270 + \frac{P}{2}\right) = 180$ --------- (2)

From (1) and (2) we get

$P = +16\ \frac{4}{11}$ (possible) and $P = 81\ \frac{1}{11}$. $P = 81\ \frac{1}{11}$ is not possible the time is between 9 O' clock and 10 O' clock

Hence required time is $9 : 16\ \frac{4}{11}$ O' clock.

5.33. At what time between 1 O' clock and 2 O' clock do the hands of a clock coincide with each other?

Sol: To coincide with each other, the angle between them should be 0°. Let this happen P minutes after 1 O' clock. We have the angle made by the hour hand with the vertical line (12 O' clock position of the hands) as 30°at 1 O' clock.

Hence, $\left(30 + \frac{P}{2}\right) - 6P = 0$

$\Rightarrow P = 5\frac{5}{11}$ minutes

Hence the two hands will coincide at

$1:05\ \frac{5}{11}$ O' clock.

5.34. In 12 hours, how many times do the hands of a clock make an angle of 180°?

Sol: Between any two hours including the end times, the two hands of the clock make an angle of 180° for one time. In 12 hours the hands of the clock make an angle of 180° only 11 times, since, between 5 O' clock and 7 O' clock, the hands of the clock will be on a straight line for only one time i.e., at 6 O' clock.

5.35. If the hands of a clock coincide every 64 minutes, then how much time does the clock gain or lose per day?

Sol: A clock which shows the correct time has its hands coinciding every $65\frac{5}{11}$ minutes. If the hands of a clock coincide every 64 minutes, the clock takes $65\frac{5}{11}$ minutes of correct time.

Hence the clock gains $65\frac{5}{11} - 64$ i.e $\frac{16}{11}$ minutes every 64 minutes.

Hence in one day it gains s $\frac{(24)(60)\left(\frac{16}{11}\right)}{64}$

$= \frac{360}{11} = 32\frac{8}{11}$ minutes.

Concept Review Questions

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Gopal travelled the first half of his journey time at an average speed of 40 kmph and the remaining half at an average speed of 50 kmph. Find his average speed (in kmph) for the journey.

2. A car left A for B at an average speed of 60 kmph and reached B one hour late. If it had left at an average speed of 80 kmph, it would have reached B one hour early. Find the average speed at which the car must travel to arrive at B on time.

(A) 70 kmph (B) $\frac{500}{7}$ kmph

(C) $\frac{480}{7}$ kmph (D) $\frac{440}{7}$ kmph

3. While travelling from office to home, Alok's car developed a problem so he took 25% more than the usual time to reach home. His speed in this case is what fraction of the usual speed?

(A) $\frac{3}{4}$ (B) $\frac{4}{5}$

(C) $\frac{3}{5}$ (D) Cannot be determined

4. If a man travelled at 3/4th of his usual speed, the time taken is how many times the usual time?

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$

(C) $\frac{3}{5}$ (D) Cannot be determined

5. The ratio of the speeds of P, Q and R is 3 : 4 : 6. Find the ratio of the times that P, Q and R would take to run a certain distance.

(A) 4 : 3 : 1 (B) 2 : 3 : 4
(C) 3 : 2: 4 (D) 4 : 3 : 2

6. The ratio of the speeds of A and B is 3 : 7. If B takes 20 minutes less than A to cover a certain distance, what is the time taken (in minutes) by A to cover the distance?

7. Travelling at three-fifths of his usual speed, a man is late by 20 minutes. What is the usual time (in minutes) taken to cover the same distance?

8. Murali travelled from city A to city B at a speed of 40 kmph and from city B to city C at 60 kmph. What is the average speed of Murali from A to C given that the ratio of the distances AB and BC is 2 : 3?

(A) 48 kmph
(B) 50 kmph
(C) 52 kmph
(D) 56 kmph

9. A person covered the distance from P to Q at a speed of 40 kmph. He covered three-fifths of the distance in two-thirds of the total time. At what speed (in kmph) should he travel to complete the remaining part of the journey in the remaining time?

10. A man covered one-fourth of the total distance at 16 kmph and the remaining distance at 24 kmph. What is his average speed for the total distance?

(A) $21\frac{2}{3}$ kmph (B) $21\frac{1}{3}$ kmph

(C) $22\frac{1}{3}$ kmph (D) $22\frac{2}{3}$ kmph

11. A man misses his bus by 40 minutes if he travels at 30 kmph. If he travels at 40 kmph, then also he misses the bus by 10 minutes. What is the minimum speed (in kmph) required to catch the bus on time?

12. Haritha leaves from a certain point at 8:00 a.m. at a speed of 25 kmph. Karuna leaves from the same point at 9:30 a.m. at a speed of 37.5 kmph in the same direction as Haritha. At what time and how many kilometres from the starting point do they meet?

(A) 12:45 p.m., 122·5 km
(B) 12:30 p.m., 122.5 km
(C) 12:30 p.m., 112.5 km
(D) 12:45 p.m., 112.5 km

13. X and Y are 400 km apart. At 7:00 a.m., buses A and B left X and Y simultaneously for Y and X. If the speeds of A and B are 30 kmph and 70 kmph respectively, at what time will they meet?

(A) 11 : 30 a.m.
(B) 10 : 30 a.m.
(C) 12 noon
(D) 11 : 00 a.m.

14. Car A left P for Q at 8:00 a.m. While car B left Q for P at 9:00 a.m. If the speeds of A and B are 60 kmph and 40 kmph respectively and the distance between P and Q is 360 km, at what time will they meet?

(A) 11:00 a.m. (B) 12 noon
(C) 1:00 p.m. (D) 12:30 p.m.

15. A started running towards South at 7:00 a.m. and B started running towards South at 11:00 a.m. At what time will they meet if the speeds of A and B are in the ratio 3 : 7 and they started from the same point?

(A) 1 p.m.
(B) 2 p.m.
(C) 12:30 p.m.
(D) Cannot be determined

16. Ajay left X for Y at 9:00 a.m. At the same time, Bala left Y for X. After their meeting at a point on the way, Ajay took 16 minutes to reach Y and Bala took 9 minutes to reach X. Find their meeting time.
(A) 9:06 a.m.
(B) 9:09 a.m.
(C) 9:12 a.m.
(D) Cannot be determined

17. Cars X and Y started from A with speeds in the ratio 4 : 5 at 9:00 a.m. and 10:00 a.m. respectively. If both the cars travelled in the same direction, find the ratio of the distances that X and Y would have covered before meeting.
(A) 1 : 1 (B) 2 : 1
(C) 1 : 2 (D) 3 : 2

18. A train 400 m long travels at a speed of 36 kmph. Find the time (in seconds) it would take to cross a platform 600 m long.

19. Two trains have lengths of 500 m and 600 m. They are running on parallel tracks in opposite directions. Find the total distance travelled (in m) by the trains from the time they start crossing each other to the time they completely cross each other.
(A) 100 (B) 1100
(C) 2200 (D) Cannot be determined

20. In the above question, if the trains are moving in the same direction, find the distance that the faster train should travel to cross the slower train completely. (in m).
(A) 100 (B) 1100
(C) 2200 (D) Cannot be determined

21. In question 19, if the two trains are moving in the same direction, from the moment the faster train started to overtake the slower train, what is the distance it has to gain over the slower train to overtake it completely?
(A) 100 m
(B) 1100 m
(C) 2200 m
(D) Cannot be determined

22. A man rows 22 km upstream in 4 hours and 45 km downstream in 6 hours. In 10 hours, how much more distance (in km) can he row downstream than the distance he can row upstream?

23. Anand can row his boat in still water at a speed of 6 kmph. If the speed of the stream is 4 kmph, how long (in hours) will he take to row a distance of 30 km downstream?

24. Rajesh takes 4 hours to row his boat 30 km downstream in a river and 8 hours to return. Find the ratio of the speed of his boat in still water and the speed of the stream.

25. A boat takes 4 hours to cover 8 km upstream and 2 hours to cover 20 km downstream. Find the speed of the boat in still water and the speed of the stream respectively.
(A) 4 km/hr, 6 km/hr
(B) 6 km/hr, 4 km/hr
(C) 8 km/hr, 6 km/hr
(D) 6 km/hr, 8 km/hr

26. Ganesh and Girish are running along a circular track 600 m long. If their speeds are 8 m/sec and 4 m/sec respectively, how many more rounds than Girish will Ganesh complete in an hour?

27. Ajay and Vijay start running simultaneously from a point on a circular track 1000 m long at speeds of 5 m/sec and 3 m/sec respectively. Find the time (in seconds) taken by them to meet for the first time, if they are running in
(i) the same direction.

(ii) opposite directions.

28. Three cyclists start cycling simultaneously from the same point on a circular track 900 m long in the same directions at speeds of 10 m/sec, 20 m/sec and 15 m/sec respectively. How long will they take (in seconds) before they meet for the first time?
(A) 90 (B) 180 (C) 360 (D) 45

29. In question 28, how long will the cyclists take (in seconds) to meet at the starting point for the first time?

30. P and Q started running from a point on a circular track 800 m long at speeds of 6 m/sec and 2 m/sec simultaneously in opposite directions. After meeting for the first time, they exchange their speeds. After how many seconds will they meet again?
(A) 200
(B) 100
(C) 300
(D) Cannot be determined

31. Two persons P and Q are running around a circular track of length 1.8 km at speeds of 30 m/minute and 60 m/minute respectively. Both of them start at the same point in the same direction but Q reverses his direction after every 60 minutes. After how much time will they meet for the first time (in minutes)?

32. P beats Q by 125 m in a kilometer race. Find Q's speed (in m/sec) if P's speed is 1.6 m/sec.

33. In a 100 m race, A beats B by 10 m and B beats C by 10 m. By what distance does A beat C (in m)?

34. In a 800 m race, A gives B a start of 100 m. Find the ratio of the speeds of A and B if they finish simultaneously.

(A) 8 : 7 (B) 9 : 8
(C) 1 : 1 (D) 3 : 2

35. In a 200 m race, A gives B a start of 10 m and beats him by 10 m. Find the ratio of the speeds of A and B.

(A) 3 : 5 (B) 5 : 3
(C) 10 : 9 (D) 9 : 10

36. A gives B a start of 10 m or 2 seconds in a 100 m race. Find A's speed if both finish simultaneously.

(A) $3\frac{1}{3}$ m/sec

(B) $5\frac{5}{9}$ m/sec

(C) $6\frac{5}{9}$ m/sec

(D) $4\frac{1}{9}$ m/sec

37. A beats B by 10 m or 2 seconds in a 100 m race. Find A's speed.

(A) $3\frac{1}{3}$ m/sec (B) $6\frac{5}{9}$ m/sec

(C) $5\frac{5}{9}$ m/sec (D) $4\frac{1}{9}$ m/sec

38. Find the angle (in degrees) between the hands of a clock at 2:30 p.m.

39. Find the time interval (in minutes) between the consecutive times that the hands of a clock are at right angles to each other.

(A) $33\frac{7}{11}$ (B) $34\frac{6}{11}$

(C) $32\frac{8}{11}$ (D) $35\frac{8}{11}$

40. Find the number of times that the hands of a clock are 180° apart in a day.

Exercise – 5(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. A bus covered a distance of 160 km in 4 hours, covering a part of it at 30 km/hr and the remaining at 70 km/hr. For how much time did the bus travel at 70 km/hr?
(A) ½ hour (B) 1 hour
(C) 1½ hours (D) 2 hours

2. Pandu takes 2 hours more than Bunty to cover a certain distance. If he doubles his speed, he will take 3 hours less than Bunty to cover the same distance. Find the number of hours taken by Bunty to cover the distance.
(A) 8 (B) 6 (C) 7 (D) 5

3. Rajat had covered one-third of the total distance of his trip when his scooter failed. He then parked it and covered the remaining distance by foot. The time for which he walked was twenty times the time for which he rode his scooter. What is the ratio of his riding speed to his walking speed?

4. The average speed for an entire journey is 60 km/hr without considering the stoppages. When the stoppages are considered, the average speed becomes 48 km/hr. How many minutes per hour on an average were the stoppages?
(A) 10 minutes (B) 12 minutes
(C) 16 minutes (D) 18 minutes

5. A person covered the first 40 km of his journey at 50 km/hr, the next 70 km at 35 km/hr and the remaining distance in 12 minutes. If the average speed for the entire journey is 52 km/hr, then find the distance covered in the last stretch of the journey (in km).

6. A parachutist, before he opens his parachute, falls for a time t_1 and covers a distance of $5t_1^2$ and after he opens his parachute, he falls for a time t_2 and covers a distance Vt_2. V is the velocity attained just before the parachute is opened and is given by $5t_1$. After how much time (in seconds) did he open the parachute, if the total distance covered by the parachutist is 1500 m and the total time taken is 30 seconds?

7. A car travels a total distance of 150 km. After travelling a part of the distance without any trouble, the car develops an engine problem and proceeds at 2/3rd of its former speed and arrives at the destination 48 min late. Had the problem developed 24 km further on, the car would have arrived 12 min sooner. Find the original distance it travelled without any problem and the speed over that part of the journey.
(A) 100 km, 60 kmph (B) 48 km, 36 kmph
(C) 72 km, 50 kmph (D) 54 km, 60 kmph

8. A starts from X and goes towards Y. At the same time, B starts from Y and goes towards X. Once A reaches Y he turns back and returns to X. Once B reaches X, he turns back and returns to Y. Throughout A travels at 54 kmph and B travels at 72 kmph. By the time they meet for the second time, B covers 36 km more than A. Find the distance between X and Y.
(A) 144 km (B) 72 km
(C) 126 km (D) 84 km

9. A flight which leaves city P at 3:00 a.m. reaches city Q at 4:00 p.m. on the same day. Another flight which leaves Q at 7:00 p.m. reaches P at 3:00 p.m. on the next day. If both flights travel non-stop at 700 km/hr and all times mentioned are local times, find the distance between P and Q (in km).
(A) 11,550 (B) 10850
(C) 6,650 (D) 5,775

10. Two men simultaneously left two places A and B. One of them left A for B and the other left B for A. Both travelled each with his own uniform velocity. Having arrived at their destinations, they turned back without stopping and proceeded to their starting points. First time, they met on their onward journey 18 km from B; the second time on their return journey 9 km from A. Find the distance between A and B.
(A) 30 km (B) 45 km
(C) 60 km (D) Cannot be determined

11. The distance between two stations P and Q is 120 km. A train starts from P to Q at 9 a.m. with a speed of 40 kmph while another train starts from Q to P at a speed of 40 kmph at 10 a.m. After one hour, the first train halts for 15 minutes at a station before it continues the journey while the second train does not halt anywhere. At what time do the two trains meet?
(A) 11:00 a.m. (B) 11:15 a.m.
(C) 11:07:30 a.m. (D) 11:12:30 a.m.

12. A police patrol party travelling at 60 kmph crosses an escaping thief travelling in the opposite direction at 48 kmph. The police party has to travel for a further 5 minutes before it can find a gap in the median where it can take a U turn and start chasing the thief. After how much time (in minutes) after the police party crosses the thief does it catch him?

13. Two bombs were exploded at a place P with a time interval of 40 minutes. A person moving away from P heard the first explosion at a point A and the second explosion when he was at a point B. If he heard the explosions at an interval of 41 minutes and the speed of sound is 331 m/s, what is the distance between A and B (in m)?

14. A train travelling at 72 kmph starts overtaking a motorcyclist travelling at 36 kmph at 4 p.m. and overtakes him in 20 seconds. Then the train travels for exactly half an hour and starts crossing another motorcyclist travelling at 36 kmph in the opposite direction. When will the second motorcyclist meet the first motorcyclist?
(A) 4:45 p.m. (B) 5:00 p.m.
(C) 5:15 p.m. (D) None of these

15. Two trains of lengths 200 m and 100 m simultaneously enter a tunnel of length 300 m from opposite ends at the same time on parallel tracks. The respective speeds of the two trains are 36 kmph and 18 kmph. After how much time (in seconds) from the instant the two trains entered the tunnel will the tunnel be free of traffic again?

16. PQ is a tunnel. A dog sits at a distance of 5/11 of PQ from P. The train's whistle coming from any end of the tunnel would make the dog run. If a train approaches P and the dog runs towards P, the train would hit the dog at P. If the dog runs towards Q instead, it would hit the dog at Q. Find the ratio of the speeds of the train and the dog.
(A) 5 : 2 (B) 16 : 5 (C) 11 : 1 (D) 34 : 3

17. A man can row a distance of 30 km downstream and return in a total of 8 hours. If the speed of the boat in still water is four times the speed of the current, find the speed of the current (in kmph).

18. A man swims from A to B and back in 4½ hours. A block of wood when allowed to go with the stream from A to B takes 6 hours. What is ratio of the speed of the man in still water to that of the stream?
(A) 2 : 1 (B) 3 : 1
(C) 4 : 1 (D) 5 : 1

19. By hiring a boat X, Rohan took 12 hours to cover a round trip between two points that are 40 km apart in a river. By hiring another boat Y, whose speed in still water is twice that of boat X, he took $3^3/_4$ hours for the round trip. Find the speed (in km/hr) of boat X in still water.

20. A person took 15 seconds to walk up an ascending escalator. He took 75 seconds to walk down the same escalator. Find the time he would take to walk up/down the escalator when the escalator is switched off (in seconds).

21. In a race of length l metres, Ram beats Shyam by a metres and Shyam beats Ghanshyam by b metres. Ram beats Ghanshyam by (in metres)
(A) $\left(a+b-\frac{ab}{l}\right)$ (B) $\left(a-b+\frac{ab}{l}\right)$
(C) $\left(ab-\frac{a+b}{l}\right)$ (D) $\left(ab-\frac{a-b}{l}\right)$

22. In a running race Sunita gives Asha a headstart of 350 m and still beats her by 50 m. If Sunita's speed is 1¼ times Asha's speed, what is the length of the race (in m)?

23. A beats B by 50 metres in a race of 500 metres. B beats C by 50 metres in a race of 1000 metres. If A beats C by 29 seconds, in a race of 500 metres how long (in minutes) will B take to run a distance of 2.7 km?

24. Three runners X, Y and Z run a race at uniform speeds. X beats Y by 12 m and beats Z by 24 m. Y beats Z by 15 m. Find the length of the race (in m).

25. Akash, Anurag and Rishab are running around a circular track of length 900 m with respective speeds of 15 m/s, 20 m/s and 30 m/s. Akash and Anurag are running in the same direction while Rishab is running in the opposite direction. After how much time will all the three of them meet for the first time?
(A) 20 seconds (B) 60 seconds
(C) 120 seconds (D) 180 seconds

26. Two points A and B are diametrically opposite points on a circular road of circumference 12 km. A cyclist started from A and made three rounds. He made the first round with a speed of 12 kmph and decreased his speed by 3 kmph after every round. What is the interval between the first time he passes through B and the third time he passes through B?
(A) 200 minutes (B) 100 minutes
(C) 85 minutes (D) 170 minutes

27. P and Q are running around a circular track of length 400 m in opposite directions with initial speeds of 10 m/s and 40 m/s respectively. Whenever they meet, P's speed doubles and Q's speed halves. After what time (in seconds) from the start will they meet for the third time?

28. There are two clocks, which are set to correct time on Sunday at 12:00 noon. The first clock gains 2½ minutes every hour while the second clock loses 1½ minutes every hour. When will they be 2 hours apart?
(A) Monday 9:00 p.m.
(B) Tuesday 12:00 mid-night
(C) Monday 6:00 p.m.
(D) Tuesday 6:00 a.m.

29. Manisha left her house between 2 O' clock and 3 O' clock in the afternoon. She returned home between 5 O' clock and 6 O' clock in the evening and noticed that the hours and minutes hand interchanged their positions with what they were when she went out. At what time did she return?
(A) 5:10 p.m. (B) $5:12\frac{8}{143}$ p.m.
(C) $5:12\frac{24}{143}$ p.m. (D) $5:12\frac{119}{143}$ p.m.

30. Find the angle (in degrees) between the hands of a clock at 5:15 p.m.

Exercise – 5(b)

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. A train covered a distance of 250 km, partly at 40 km/hr and the remaining at 60 km/hr. Find the distance covered at 40 km/hr (in km) if it took 5 hours for the train to cover the distance.

2. Rajesh had covered two-third of a certain distance when his car had a breakdown. He parked it and covered the remaining distance by foot. His time of travel by foot was 18 times his time of travel by car. The ratio of Rajesh's car's speed to Rajesh's walking speed is .

3. A man started 15 minutes late and by travelling at a speed which is $^{5}/_{4}^{th}$ of his usual speed reached his office 20 minutes early. What is the usual time of his journey (in minutes)?

4. I had to catch a bus which was 225 m ahead of me. If the speed of the bus was 2.5 m/sec and my speed was 36 km/hr, in how much time (in seconds) did I catch the bus?

5. A train crosses two bridges 370 m and 480 m long in 51 and 62 seconds respectively. Find the speed of the train (in km/hr).

6. A train takes 10 seconds to cross a man standing on a platform and 44 seconds to cross the platform. What is the length of the platform (in m) if the speed of the train is 72 km/hr?

7. A car takes 2 hours more to cover a distance of 480 km when its usual speed is reduced by 8 km/hr. Find its usual speed (in km/hr).

8. Ramu starts from P towards Q at a speed of 30 km/hr and after every 12 minutes increases his speed by 5 km/hr. If the distance between P and Q is 51 km, then how much time does he take to cover the distance?
(A) 60 minutes (B) 72 minutes
(C) 90 minutes (D) 120 minutes

9. Mr. Patel and Mrs. Patel were travelling from town A towards town E. On the way they would pass through the towns B, C and D in the same order. On reaching B, Mrs. Patel asked "How far have we come?", Mr. Patel replied, "Half the distance between the towns C and D." On reaching D, 180 km from B, she asked "How far is E from here?" Mr. Patel replied, "Half the distance between the towns B and C." Find the distance (in km) between the towns A and E.
(A) 270 (B) 210 (C) 330 (D) 360

10. X started from a point A towards point B. After 2 hours, Y started from B towards A. By the time X travelled one-fifth of the total distance, Y had also travelled the same. If Y's speed is thrice that of X's speed, find the difference in the times (in hours) taken by X and Y to reach their destinations.

11. Alok ran from P to Q at x kmph. After reaching Q, he rested for x hours. He ran from Q to R at thrice the previous speed. He rested at R for 3x hours. He ran from R to S at twice his speed from Q to R. He took a total of 30 hours for the journey. PQ = QR = RS = 36 km. If x is an integer, find the value of x.

12. P and Q are 2 stations. A single track runs between them. Train A leaves everyday from P at 6:00 a.m. and reaches Q. Another train B starts from Q as soon as A reaches Q. B reaches P at 7:00 a.m. The speed of B is four times that of A. One day, A started from P, 10 minutes late as per the schedule. In order to maintain B's right arrival time at P, both trains travel at an increased speed. If B had doubled its speed, find the ratio of the new speeds of A and B.
(A) 3 : 22 (B) 22 : 3 (C) 38 : 3 (D) 44 : 9

13. The distance from my house to my friend's house is 12 km. I walked at a speed of 4 km/hr and after every kilometre took rest for 10 minutes. How much time did it take for me to reach my friend's house?
(A) 3 hours (B) 4 hours and 50 minutes
(C) 5 hours (D) None of these

14. Amir takes 10 minutes to travel from A to B whereas Simran takes 15 minutes to travel the same distance. Amir and Simran start from A to B and B to A respectively at 8:00 a.m. At what time will they meet?
(A) 8:03 a.m. (B) 8:06 a.m.
(C) 8:09 a.m. (D) 8:12 a.m.

15. Ashish and Bali run towards each other from P and Q respectively with respective speeds of 36 kmph and 45 kmph. After meeting each other if Ashish reaches Q in 5 hours, in how many hours does Bali reach P?

16. A train of length 180 m travelling at 72 kmph overtook a motorcyclist travelling at 36 kmph at 4:00 p.m. At 5:00 p.m. it overtook another cyclist travelling in the opposite direction at a speed of 18 kmph. How much time after 5:00 p.m. will the cyclist meet the motorcyclist?
(A) 1 hour 20 minutes (B) 1 hour 30 minutes
(C) 1 hour 45 minutes (D) None of the above

17. Two cyclists simultaneously start from A to B and B to A respectively. They cross each other after a time t hours. The first person reaches B in another t_1 hours while the second person reaches A in another t_2 hours. Then

(A) $t = \frac{t_1 + t_2}{2}$ (B) $t = \frac{2t_1t_2}{t_1 + t_2}$

(C) $t = \sqrt{t_1 t_2}$ (D) $t = \frac{t_2 - t_1}{2}$

18. Car A left town x for town y at 7:00 a.m. at 54 kmph. Car B left y for x at 8:00 a.m. at 48 kmph. The distance between x and y is 90 km. Find the time when A and B would be 19 km apart for the first time.
(A) 8:10 a.m. (B) 8:06 a.m.
(C) 8:12 a.m. (D) 8:16 a.m.

19. Two stations X and Y are 330 km apart. Train A starts from X at 9:00 a.m. at 60 kmph towards Y. Train B starts from Y at 10:00 a.m. at 40 kmph towards X. At a station 120 km from Y and 210 km from X, A halts for 35 minutes. B does not halt anywhere along its journey. Find the time when both trains meet for the first time.
(A) 12:00 p.m. (B) 12:30 p.m.
(C) 1:00 p.m. (D) 1:30 p.m.

20. A policeman was travelling at 90 kmph. He crossed a thief travelling at 60 kmph in the opposite direction. He had to travel for another 6 minutes before he could find a gap in the median where he could take a U turn and start chasing the thief. After they crossed each other how long would the policeman take to catch the thief? (in minutes)

21. Two cars A and B started from towns X and Y respectively in opposite directions towards each other simultaneously. After reaching the other town they returned to their starting town using the same route. They travelled at uniform speeds throughout their journeys. They met for the first time at 30 km from Y. They met for the second time at 10 km from X. Which of the following can be the distance (in km) between X and Y?
(A) 70 (B) 80 (C) 90 (D) 60

22. Anand and Bhaskar ran towards each other starting from M and N simultaneously at speeds of 5 kmph and 4 kmph respectively. After meeting each other, Anand takes 9 hours less to reach N, than the time taken by Bhaskar to reach M. Find the distance (in km) between M and N.

23. Two men A and B started walking towards each other's starting point simultaneously from two points X and Y which are 12 km apart. They meet after 1 hour. After meeting, A increased his speed by 6 kmph. B reduced his speed by 6 kmph. They arrived at their destinations simultaneously. Find the initial speed of A (in km/hr).
(A) 2 (B) 3 (C) 4 (D) 5

24. Adam and Bryan start from P and Q towards Q and P respectively, at the same time. After they meet, exchange their speeds and proceed towards their respective destinations. If Adam takes 128 minutes to travel from P to Q, what is the time taken by Bryan to travel from Q to P?
(A) 96 minutes
(B) 112 minutes
(C) 128 minutes
(D) Cannot be determined

Directions for questions 25 and 26: These questions are based on the information given below.

Train P has a length of 300 m and a speed of 72 kmph. Train Q has a length of 600 m and a speed of 90 kmph. Both enter a tunnel in opposite directions simultaneously. The length of the tunnel is 600 m.

25. Which train exits the tunnel first? What length of the other train is still in the tunnel at that time? (in m)
(A) P, 75 (B) Q, 75 (C) P, 90 (D) Q, 90

26. Find the distance between the point where the rear ends of the trains cross each other and the point of entry of the slower train (in m).
(A) $398\frac{1}{3}$ (B) $366\frac{2}{3}$ (C) $216\frac{2}{3}$ (D) $233\frac{1}{3}$

27. A train is 270 m long. It overtook a motorcyclist travelling at 36 kmph in 27 seconds. An hour later, it overtook a cyclist in 18 seconds. How long after the train overtook the cyclist would the motorcyclist overtake it approximately? (in minutes)
(A) 120 (B) 210 (C) 180 (D) 150

28. A train, 180 m long, crossed a 120 m long platform in 20 seconds, and another train travelling at the same speed crossed an electric pole in 10 seconds. In how much time will they cross each other when they are travelling in the opposite direction (in seconds)?

29. A train travelling at 36 km/hr takes 48 seconds to cross a bridge. It then crosses a man cycling at 9 km/hr in the same direction in 20 seconds. Find the length of the bridge (in metres).

30. The speed of a boat in still water is 6 kmph. What is the speed of the stream (in kmph) if the boat can cover 32 km downstream and 16 km upstream in the same time?

31. A man can row 30 km downstream in 3 hours 45 minutes, and 11 km upstream in 2 hours 12 minutes. The speed of the man in still water and the speed of the stream (both in kmph) are _____ respectively.
(A) 6 and 2
(B) 6.8 and 1.8
(C) 6.5 and 1.5
(D) 7 and 3

32. Two persons A and B are running around a circular track of length 1800 m at speeds of 40 m/minute and 50 m/minute respectively. Both of them start simultaneously at the same point in the same direction but B reverses his direction every time he completes one round. After how much time (in minutes) will they meet for the first time?

33. Rohan and Sohan started running simultaneously in opposite directions from a point on a 900 m long circular track at 15 m/sec and 10 m/sec respectively. Whenever they meet, they exchange their speeds and continue to travel in the same directions. Find the shortest distance (in m) between Rohan and Sohan along the track when Rohan completes $4^1/_2$ rounds.
(A) 100 (B) 125 (C) 150 (D) 175

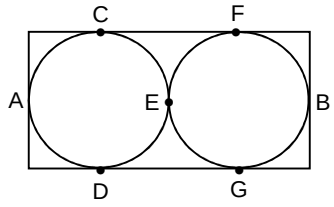
34. Ravi and Vikram ran a 9 km race on a circular track of length 900 m. They started running simultaneously from the same point on the track. They completed one round in 9 seconds and 15 seconds respectively. After what time from the start will the faster person meet the slower person for the last time (in seconds)?

35. In a 1000 m race A reaches the finishing line 5 seconds earlier than B and beats B by 50 m. What is A's speed?
(A) 10 m/s (B) $10^{10}/_{19}$ m/s
(C) $9^9/_{19}$ m/s (D) $11^9/_{19}$ m/s

36. In country C, clocks are manufactured in a special way. The total area covered by the hours hand in 4 days is 3/16th of the total area covered by the minutes hand in one day. Find the ratio of the lengths of the minutes hand and hours hand.
(A) 3 : 4 (B) 4 : 3 (C) 9 : 16 (D) 16 : 9

37. At what time between 1 and 2 O' clock are the two hands coincident?
(A) $1 : 12^5/_{11}$ (B) $1 : 07^5/_{11}$
(C) $1 : 05^5/_{11}$ (D) $1 : 08^5/_{11}$

38. The diagram below shows a jogging park, which has two identical circular tracks touching each other. A rectangular track encloses the circles. Its edges are tangential to the circles.



P and Q are two friends. They start jogging simultaneously from the points A and B respectively. P jogs along a path in the shape of a eight i.e., ACEGBFEDA. Q jogs along the rectangular track. Both reach their starting points simultaneously. By what percentage, is the distance run by Q less than that run by P?
(A) $4^1/_{11}\%$ (B) $4^2/_{11}\%$ (C) $4^6/_{11}\%$ (D) $4^4/_{11}\%$

39. A car travelled a distance of 900 km. It developed an engine problem after travelling for some distance. It travelled the remaining distance at 3/5th of its speed. It reached 2 hours late. If the engine problem had developed after it had travelled for another 150 km, it would have reached 1 hour earlier than the time it actually reached. Find the distance it travelled without any problem and its speed over that part of the journey. (in km, in kmph).
(A) 500, 100 (B) 400, 75
(C) 600, 100 (D) 700, 75

40. Mahesh and Naresh started simultaneously from a point on a circular track in opposite directions. The radius of the track is 7 m. After every meeting, they exchange their speeds as well as directions. If Mahesh starts with a speed 4 times that of Naresh, then find the total distance that Naresh would have covered when he met Mahesh for the 22nd time. (in m) $\left(\text{Take } \pi = \frac{22}{7}\right)$.

41. In a 500 m race, A beats B by 40 m. In a 1000 m race, B beats C by 40 m. If A beats C by 14·6 seconds in a 500 m race, find the time taken (in minutes) by B to run 2.4 km.

42. Ajay started a test at a certain time between 5 O' clock and 6 O' clock and ended between 6 O' clock and 7 O' clock. Ajay observed that the minutes and hours hands when he ended the test had interchanged their positions with those when he started. How much time did he take for the test?
(A) $52^3/_{11}$ minutes (B) $55^5/_{13}$ minutes
(C) $47^4/_9$ minutes (D) 45 minutes

Directions for questions 43 to 45: These questions are based on the information given below.

There are 3 stretches in a race – X, Y and Z, each of which is 4 km long. The following table shows the mode of coverage of each stretch and the maximum and the minimum speeds of covering them.

Stretch	Coverage Mode	Minimum Speed (in kmph)	Maximum Speed (in kmph)
X	Bicycle	20	25
Y	Motorcycle	40	50
Z	Car	50	75

Each stretch is covered at a constant speed. The previous record in completing the race was 22 minutes.

43. Ajay travelled at the minimum speed by bicycle as well as motorcycle. If he did not break the previous record, his speed (in kmph) over stretch Z cannot exceed.

44. Ajay travelled stretch X taking the maximum possible time. The time taken for him to cover stretch Z is $66^{2}/_{3}$% less than the time taken to travel stretch X. If he took $4^{6}/_{11}$% less time to complete the race than the previous record, then find the speed at which he has travelled stretch Y (in kmph).
(A) 42 (B) 44 (C) 46 (D) 48

45. Ajay's overall average speed is 450/11 kmph. His speed over stretch X was $5/12^{th}$ of his average speed over the stretches Y and Z together. Find his speed over stretch X (in kmph).

Directions for questions 46 to 55: Each question is followed by two statements, I and II. Indicate your responses based on the following directions:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using statements I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using statements I and II together.

46. Prasanna travels from A to B and back. What is his average speed for the whole journey?
I. Prasanna totally travels 200 km in a span of 6 hours.
II. Prasanna's speed from A to B is 30 km/hr while his speed during the return journey goes up by $33\frac{1}{3}$ % when compared to the forward journey.

47. What is the ratio of the speeds of Pulkit and Angadh?
I. Even if Pulkit starts half an hour earlier than Angadh he takes one hour more than Angadh in covering a certain distance.
II. Angadh and Pulkit start from the two ends of a track at the same time. After crossing each other Angadh and Pulkit take $3^{1}/_{3}$ hours and $4\frac{4}{3}$ hours to reach opposite ends respectively.

48. What is the speed of the train?
I. To cover a certain distance in 2 hours, Karun travelled 12 km by ship, 45 km by train and 6 km by rickshaw.
II. The ratio of the speeds of the ship, the train and the rickshaw by which Karun travelled is 4 : 15 : 1.

49. Aarti and Bhallu participated in a race where they have to run from P to Q and back to P. Bhallu being the faster of the two, reached Q first and met Aarti at R on his way to P. What is the length of PR?
I. PQ = 20 km
II. Ratio of speeds of Aarti and Bhallu is 3 : 5

50. A train crosses two men moving in the same direction. The speed of one man is 2 m/s more than that of the other. What is the length of the train?
I. The train crosses the slower man in 9 seconds and the other man in 10 seconds.
II. The faster man walks at 4 m/s.

51. What is the speed of the train?
I. It takes 6 seconds to cross a stationary observer.
II. It takes 25 seconds to cross a 750 m long platform.

52. What is the time taken to travel from A to B at x kmph?
I. Time taken to travel from A to B at x kmph is 2 hours more than the time taken at (x + 10) kmph.
II. Time taken to travel from A to B at x kmph is 2 hours less than the time taken to travel from A to B at 3x/4 kmph.

53. By what distance does Asif beat Alok in a 800 m race?
I. In a 400 m race, Biswas beats Alok by 30 m.
II. In a 800 m race, Biswas beats Asif by 40 m.

54. At what time between 3 O'clock and 4 O'clock is the angle between the hands K°?
I. K = 130.
II. K = 100.

55. Speed of a motorboat in still water is how many times the speed of the water current?
I. The time taken by the boat to cover a certain distance down stream is half the time taken by it to cover the same distance upstream.
II. The speed of the boat upstream is 10 m/s.

Key

Concept Review Questions

1. 45
2. C
3. B
4. B
5. D
6. 35
7. 30
8. B
9. 48
10. B
11. 45
12. C
13. D
14. B
15. B
16. C
17. A
18. 100
19. B
20. D
21. B
22. 20
23. 3
24. 3
25. B
26. 24
27. (i) 500
    (ii) 125
28. B
29. 180
30. B
31. 60
32. 1.4
33. 19
34. A
35. C
36. B
37. C
38. 105
39. C
40. 22

Exercise – 5(a)

1. B
2. A
3. 10
4. B
5. 46
6. 10
7. D
8. D
9. A
10. B
11. C
12. 50
13. 19860
14. D
15. 80
16. C
17. 2
18. B
19. 12
20. 25
21. A
22. 2000
23. 17.1
24. 60
25. D
26. D
27. 26
28. C
29. C
30. 67.5

Exercise – 5(b)

1. 100
2. 36
3. 175
4. 30
5. 36
6. 680
7. 48
8. B
9. A
10. 10
11. 3
12. A
13. B
14. B
15. 3.2
16. D
17. C
18. A
19. C
20. 36
21. B
22. 180
23. B
24. C
25. A
26. B
27. A
28. 11
29. 330
30. 2
31. C
32. 40
33. C
34. 90
35. B
36. B
37. C
38. C
39. C
40. 484
41. 9.6
42. B
43. 60
44. D
45. 25
46. B
47. A
48. C
49. C
50. A
51. C
52. A
53. C
54. A
55. A